Algorithmic Variations and Measurements of the WebMA: Online Algebra System

Diem M. Nguyen
dnguyen@bgsu.edu
Department of Mathematics & Statistics
Bowling Green State University

Abstract

This article reports results of the implementation of the online algebra system, WebMA [see WebMA], into an undergraduate course that prepared pre-service teachers for their high school mathematics teaching. WebMA is a web-based system with a complete mathematical engine and algorithmic random generators that can be used to access student learning outcomes. Instructors can utilize it to diagnose students' prior knowledge and to evaluate student work during the course with various different types of assessment as well as provide immediate feedback throughout the course period. These features present numerous extraordinary benefits for instructors who are interested in using online systems to assess learning outcomes and increase student engagement. The experience that we learned based on student feedback will be conveyed.

Introduction

Recently there has been a strong surge towards the incorporation of online technology to assist teaching mathematics at all levels. Online technology tools open new opportunities and challenges in making mathematics more accessible to students. Building on recent suggestions of new principles for assessment by the congressional Web-based Education Commission [WBEC, 2001], online practices are heading in the direction of "[engaging] students in learning and requiring thinking skills" [McMillan, 2001, p. 14]. Online practice allows students to have independent drill and self-evaluation, and helps students improve computational skills and problem solving (see Morgan & O'Reilly, 2001). Online practice with random features allows for the repetition of important concepts, keeps the students on task throughout the semester, and provides timely feedback to students as well as instructors. While students have more practice and receive timely feedback for improvement, the instructors can save time in grading and can gather an enormous amount of the invisible information and assessment results that help them revisit concepts that need more clarification in a timely manner [see Sanchis, 2001].

According to Charman and Elmes [1998], immediate feedback is considered the most important issue and the strongest asset in the online learning and practice, especially in relation to problem solving. With a traditional way of assessment, feedback faces serious limitations. Finding every student's errors and writing appropriate and effective comments on different methods of problem solving are usually difficult and time consuming. Moreover, receiving the comments on work that was done a week or even longer before is not as motivating for students since they forget the processes that they went through in working out these problems. Therefore, those comments (if ever read) may be no longer helpful at that point. Contrary to the traditional practice, the online practice can affordably provide meaningful and immediate feedback within the assessment process [Nguyen, Hsieh & Allen, 2006]. Bransford, Brown and Cocking [1999] and Polya [2004] agree that immediate feedback not only enhances the understanding but also increases self-evaluation and self-motivation. According to Bennett [2001], Kinzer et. al [2004] and Salpeter [2003], online assessment not only serves as summative but also as formative information for teaching improvement.

The instructors can integrate online practice with automated and adapted instruction to embrace the learning process for both inside and outside of the classroom. McCabe's study in 2002 conveyed that online assessment helps instructors to find ways to effectively utilize practicing tasks to identify overall achievement patterns as well as to help each individual student learn. With its outstanding features, online practice can be considered a mind-tool that "drives and shapes students' learning", achievement, habit and motivation, and aids teachers' instruction and evaluation [Morgan & O'Reilly, 2001, p. 185]. Online practice, according to Allen [2001], constitutes an integral part of the mathematics curriculum and the learning process.

Purpose of the study

Typical first year college or university course leveling and advising for the enrollments of thousands of incoming students is a labor intensive and difficult task. Simultaneously, with a strong commitment to a student-centered environment that offers multiple opportunities for students' success, the ability to offer various forms of frequent and convenient assessment and feedback is the main objective for the implementation of WebMA and for this study. In this article, the special features of WebMA and its ability in carrying out the mission of leveling incoming students using an online placement exam and providing students with frequent alternative assessment and multiple independent practices to enhance mathematics learning is discussed. Numerous extraordinary benefits such as the ability to administer a large-scale test, the flexibility of access time or location, the opportunity for repeated self-testing and practice tailored with instant feedback will be discussed. Some experiences with mathematical difficulties and limitations will be illustrated along with possible suggestions for improvement.

Special features of WebMA

WebMA is a web-based assessment system used to administer graded tests and homework with mathematical content. Recently there has been no shortage of online assessment that is available with textbook publishers providing multiple-choice question banks that are specific to the material being covered. For example, MapleTA is an online assessment tool that is available for multiple formats of testing questions and requires an annual server support fee depending upon the number of users [see MapleTA]. However, unlike some other online systems, three distinguished features of WebMA are (1) its available syntax for creating not only multiple-choice, but also true/false, matching and short answers with a complete mathematical engine to perform computations and provide the visualization of mathematical formulas; (2) its ability in creating question banks with algorithmic random generators for diagnostic testing and mastery practice along with its facilities to assign partial credit; and (3) its ability to "communicate" with the university course management system software for data parsing and student authentication. Apart of all these features, WebMA stands out as a non-commercial software.

WebMA is an Internet-based software. It can be accessed via any mainstream browser. Particularly, it is accessible without requirement of any plug-in components or special software on students' computers. It only requires the host server to have Maple software installed. Once it is set-up, all administrative tasks are performed via the Internet including authoring quizzes, obtaining grade sheets, and providing statistical analysis of the variety of answers. For testing or practicing purposes, there are several options, depending on whether a quiz is used as formative or summative assessment. These options include guest access, multiple randomized versions, level of feedback, availability of solutions, time-length of completion, due date, and quiz weight for each repeated approach. To create quizzes, some knowledge of Maple and LaTeX is necessary; but there is neither experience with HTML nor programming skills required. However, to take quizzes with short-answer questions, students are required to know some basic Maple syntax.

The following paragraph provides samples of question creation, output, answer entering, validation and grading:

1. Sample of a multiple-choice question with solution:

- Question coding:

```
t> What is $@Limit(cos(1/x),x=infinity)@$?
c> MC(0,1,-1,+infinity,-infinity)
a> 1
sb>
```

- Solution coding:

```
t> As x-->$@infinity@$, we have $@(1/x)@$-->0, where $@Limit(cos
(1/x),x=infinity)@$ = cos(0) = 1
end>
```

- Question display:

Question 5 Previous 1 2 3 4 5 6 7 8 9 10 Next Validate

What is $\lim_{x\to\infty}\cos(x^{-1})$?

A ○ 0

B ○ 1

C ○ -1

D ○ ∞

E ○ $-\infty$

- Solution display:

Solution:
As x->∞, we have x^{-1}->0, where $\lim_{x\to\infty}\cos(x^{-1}) = \cos(0) = 1$

2. Sample of a short answer question with automatic grading:

- Question coding:

```
t> Give an example of an even function.
ap> $f(x):=$
s> [(ans)->`aim/Test`(ans-subs(x=-x,ans),0),x^2]
end>
```

- Question display:

Question 6 Top 1 2 3 4 5 6 Bottom Validate

Give an example of an even function.
f(x):=

- Students' answers and grading:

  - Sample answer 1 and its grading:

Question 6 Top 1 2 3 4 5 6 Bottom Validate

Give an example of an even function.
f(x):=3*x^2+5
Your last answer was:

$$3\,x^2+5$$

Your answer is correct.
The mark for your last attempt was 1.00

  - Sample answer 2 and its grading:

Question 6 Top 1 2 3 4 5 6 Bottom Validate

Give an example of an even function.
f(x):=2*x^4-x^2+x-1
Your last answer was:

$$2\,x^4-x^2+x-1$$

Your answer is not correct.
Your mark for your last attempt was 0.00.

3. Sample of a question with Maple graphic:

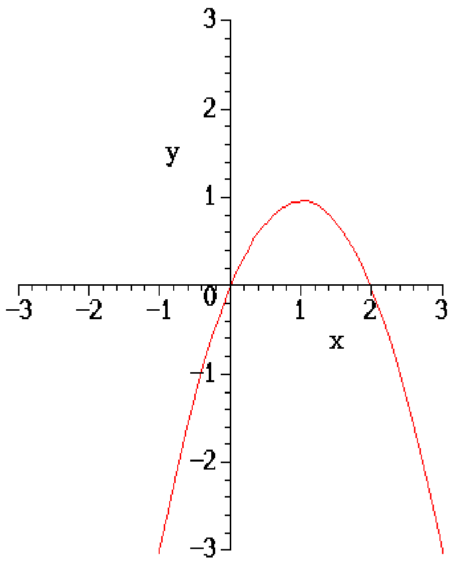
- Question coding:

```
h> f_:=-(x-1)^2+1;
   J:=plot(f_,x=-3..3,y=-3..3);
t> For the function f(x) whose graph is the parabola sketch in the
   following figure:
p> J
t> Find the value $@x@$ where $@f(x)@$ has the maximum height.
ap> $x:=$
a> 1.0
end>
```

- Question display:

Question 7 Top 1 2 3 4 5 6 7 Bottom Validate

For the function f(x) whose graph is the parabola sketch in the following figure:



Find the value x where f (x) has the maximum height.

x=

Incorporating WebMA into learning and assessment

WebMA in diagnostic entry testing. At the experimental university site, along with ACT and SAT scores, diagnostic Algebra and Calculus readiness tests are administered to all new coming students to identify the strengths and weaknesses in students' prior knowledge for the purpose of course leveling and advising. The tests were comprised of 35 multiple-choice questions ranging from arithmetic operations to trigonometric evaluations. The algorithmic variation was not used for the diagnostic test, but optionally the variation of question orders within each category of arithmetic, algebra and trigonometry allowed the system to provide each student with a different version of the test. The test was one hour long. Each student could login to take the test by using his or her provided unique password. If, for any reason, the Internet connection was lost during the test-taking period, the students' answers would be automatically saved with the time stamped for the next continuous attempt. This additional feature makes the WebMA fitted well into the online testing environment and outstanding in comparison with other current existing online testing systems. It is necessary to mention that unless the Internet connection was lost, each student was allowed to access the test only one time.

A complete database system allowed the university to collect, gather and analyze test data for over 4,500 students every summer with minimal effort. Simultaneously, advisors could access students' test scores, obtain knowledge-based information, and provide course leveling suggestions immediately after the students finished taking the test. With this online system, prospective students have more freedom to take the test at a more convenient place and time for them and prior to their meeting with an academic advisor for course placements. Overall, the use of WebMA has confirmed the ultimate effects of the online technology in large-scale testing and fast-tracked reporting system prospected by Bennett [2001] and Phipp [2004].

WebMA in weekly tutoring and quizzes. Beyond the diagnostic and placement purposes, the summary data analyses on students' strengths and weaknesses prior to entering the course provided instructors with some foresights to students' primary areas of difficulties. These allowed instructors, as well as the department, to arrange suitable reviewing and tutoring sessions. With Maple as a mathematical engine, an extraordinary feature of WebMA is its ability to assign and grade short-answer questions with possible partial credits. Therefore, besides serving as the online diagnostic test for new coming students, WebMA has been utilized in both online and classroom teaching in various courses. It was particularly used to enhance mathematics learning and practices for pre-service high school mathematics teachers.

Specifically, the instructor of the course used WebMA to create everyday pop quizzes, and additional mastery quizzes for practice and self-review. These quizzes included multiple-choice, true/false, matching, and short-answer questions. The questions were designed with algorithmic variations incorporated with automated answers and solutions. Each student needed a password to get in the system to take the quizzes. If it was a mastery practice quiz, students could practice it as many times as they desired, and scores were not recorded. Answers and solutions for each practice quiz could be viewed at any time. If it was a pop quiz, students were allowed to take it only one time, and answers and solutions could only be viewed after the quiz had been submitted.

In order to enter the short answer responses, students needed to be familiar with Maple syntax. Therefore, before taking the first practice, students were instructed with basic Maple procedures on how to get help from Maple, how to correct syntax errors, and how to compare a correct answer entering with an incorrect one. It is worthwhile to mention that WebMA does provide a systemic syntax check-up for each short-answer entering.

Overall experience and student feedback

The online diagnostic testing system has completely substituted the paper-and-pencil tests since 2004 at this experimental university. Besides lending the flexibility of access time, location and advising process, the online diagnostic testing system facilitates large-scale data information, collecting, integrating, restoring and circulating among a large size university population. Data from item analysis inform faculty of patterns of common difficulties as well as the strong and weak strands of new coming students in each year. Consequently, appropriate tutoring sessions were organized focusing on helping remedial students reinstate mathematical background necessary for them to enter the university mathematics level and assure their retention and success in the course that they were placed in.

Of the weekly tutoring and quizzes, this new online technology provided extraordinary benefits to students and instructors in classroom teaching and independent practices. First of all, the opportunity to administer a pop-quiz with two or three questions at the end of each lecture without the effort of grading allowed the instructors to get immediate feedback from students. Pop-quizzes helped the instructors to evaluate student levels of understanding and identify the difficulties and misunderstandings that were crucial for revising the difficult concepts as necessary in the following class time. It also helped the instructor create appropriate review worksheets and effectively plan for tutorial activities. For students, pop-quizzes at the end of each lecture motivated them to actively engage in learning the material and enhanced their participation in questions and answers during lecture. Furthermore, taking the pop-quiz at the end of class helped students summarize the main contents of the lecture. Consequently, it helped reduce the number of daily absentees and decrease the probability that students walked out of the classroom without knowing what they had learned and had been expected to know from the lecture. Secondly, the main purpose of implementing WebMA into classroom teaching is providing students with alternative practices with automatic solutions and feedback to review for in-class quizzes and exams. There were online mastery practice sets (MPS) implemented for each quiz and exam. Problems included in each MPS were selectively extracted and generated from homework assignments, and each problem was tailored with detailed feedback and solution. Algorithmic variations were applied to allow students to have multiple practices without repeating the same problem. The variations included numerical, functional, and word changes. Each MPS was available for practice two days before each in-class written quiz or exam was given. All online tasks were available for students to review and practice for the final exam.

One benefit of the randomized feature was to allow students to be exposed to different question settings of each mathematical concept. Another benefit was a reduction in the frequency of impersonation of one student by another while taking pop quizzes. Consequently, the online daily pop quizzes and MPS had some positive effects on students' retention, exams and overall scores. In comparison with three other regular sections (RS) with 69 students of the course that did not use the WebMA, 81 students in three sections with the use of WebMA had higher levels of passing and a lower level of dropping rates. Particularly, the average passing rate for all six sections was 82%, while the passing rate for the WebMA group was 89%. From the WebMA group, there were two students who dropped out from the course and seven students who did not pass the course. From the RS group, five students dropped out and nine students did not pass the course. The following table summarizes the results of the overall scores and letter grades:

Group	Number of Students	Grade of A's (Percentage Number)	Grade of B's	Grade of C's	Grade of D's	Grade of F's and/or Dropped out
RS	69	18% (12)	38% (27)	16% (11)	7% (5)	20% (14)
WebMA	81	24% (19)	41% (32)	19% (15)	5% (4)	11% (9)

Though there were no significant statistical differences on regular exam scores between the two groups, the multivariable analysis of variance (Pillai's trace) showed a significant difference in the comprehensive final exams with five different score levels between the two groups, $F(10,195) = 2.53$, $p<0.05$. In a survey done at the end of the course, 72% (57 out of 79) of students in the WebMA responded that they felt comfortable to work with the online tasks and felt comfortable with the use of Maple, 22% (17 out of 79) of them expressed some forms of discomfort, and 6% (5 out of 79) of them particularly did not feel comfortable with the online tasks.

Following are some common feedbacks from students regarding the online practices:

1. Positive feedback:
   - I had more practice with online assignments than with regular assignments.
   - It forced me to practice more, of course, for better scores.
   - Instant feedback was important. Online practice with randomized questions provided me with more opportunities to learn independently outside of the class.
   - It was cool to know the results of the quizzes immediately.
   - I was able to verify my understanding by reading the solutions. It surely helped me a whole lot in preparing for exams.
   - Daily quizzes were extremely helpful. I often grasped the important ideas of the lecture from those quizzes.
   - Pop quizzes helped me confirm my understanding and motivated me to come to class every day.

2. Negative feedback:
   - Online practice did not allow verbal explanation.
   - Partial credit feature was not reliable.
   - The timing on the quiz was ridiculous. It made me nervous every time I took an online quiz.
   - I had to take too many quizzes through out the semester.
   - It took more time to do homework on the computer than on papers. I had to pause typing my answers sometimes in order to search for Maple commands.

Limitations

Though WebMA interfaced with a complete mathematical engine for checking answers, not all tasks could be automatically graded by the system. Only those mathematical criteria which may be checked with the computer algebra system could be implemented. It is obvious that the online algebra system was not sophisticated enough to grade verbal explanations or justifications. Also, the system could not grade the multiple-step answers but only the final step. The questions with multiple parts, such as part a and part b, had to be separated into two different questions for grading. There were some limitations particularly with the trigonometric functions; for example, the answer entering as $\cos(-x)$ would be automatically converted to $\cos(x)$. The system could not detect some abnormal forms of the question due to some negligence of the range setting for randomization. For example, the coefficient of 1 may appear in front of the variable, such as $\sum_{k=1}^{8} 1k^3$. With the short-answer responses, there were no measurement units accepted by the system. The use of symbol for any responses was specifically limited to the preset one. For example, in the sample question 6, students were requested to enter a function with x as a variable in order to receive appropriate grading from the system.

Conclusions

Throughout the study, WebMA has demonstrated that the online algebra system can assist the large-scale assessment as well as the learning and practice to improve student performance in mathematics. This web-based instrument combines the ability of online multiple-choice placement exams to facilitate large numbers of exam takers with the greater flexibility, accessibility, and additional information than the traditional testing method offers. Optional functions with the aid of mathematical engine allow the system to generate randomization of parameters for multiple problem settings. This feature allows each student to have a unique problem set for practice and unique questions for test taking. The ability to capture and grade automatically mathematical expressions and short-answer responses along with the delivery of instant feedback enrich the meaning of assessment and practice. Although some limitations still existed with the mathematical symbol manipulations and grading, WebMA provides students with independent learning opportunities. It allows them to determine their own learning outcomes through self-testing. From a pedagogical perspective, independent learning and self-testing are necessary habits for students, especially pre-service teachers, for their future success. The pre-service teachers in this study had opportunities to familiarize themselves with the use of online technology in learning. They were also aware of the existing alternative assessment devices that they can use for their future teaching and measurement processes.

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